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# The punishing factors for convex pairs are 2n-1

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## Abstract

Let Ω and ∏ be two simply connected proper subdomains of the complex plane ℂ. We are concerned with the set A(Ω, ∏) of functions f : Ω → ∏ holomorphic on Ω and we prove estimates for |f(n)(z)|, f ∈ A(Ω, Ω),z ∈ Ω, of the following type. Let λΩ(z) and λ ∏ (w) denote the density of the Poincaré metric with curvature and λ = -4 of Ω at z and of w, respectively. Then for any pair (Ω, ∏) of convex domains, f ∈ A(Ω, ∏), z ∈ Ω, and n ≥ 2 the inequality |f(n)(z)|/n! ≤ 2 n-1(λΩ(z))n/λ ∏(f(z)) is valid. The constant 2n-1 is best possible for any pair (Ω, ∏) of convex domains. For any pair (Ω, ∏), where Ω is convex and ∏ linearly accessible, f, z, n as above, we prove |f(n)(z)|/(n+1)! ≤ 2 n-2(λΩ(z))n/λ∏(f(z)). The constant 2n - 2 is best possible for certain admissible pairs (Ω, ∏)). These considerations lead to a new, nonanalytic, characterization of bijective convex functions h : Δ → Ω not using the second derivative of h.

## Keywords

Bounded functions, Close-to-convex functions, Convex domain, Convex functions, Inverse functions, Linear accessible domain, Taylor coefficients